DEFENCE R&D DÉFENSE

Optimal Fleet Replacement and Forecasting Under Uncertainty: The CP-140A Arcturus Maritime Surveillance Aircraft

Dr David Maybury

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Defence Research and Development Canada

Recherche et développement pour la défense Canada

Canada

Outline

- Aim and scope
- Model assumptions
- Methodology
- Model details
- Results
- Summary and implications

Aim and scope

- DND requested a study to help determine the most economic replacement time for a vehicle fleet
  - Decision Support
- "*In a world of intense global competition, simply setting appropriate general guidelines is not enough*."
  (Hopp and Spearman, Factory Physics)

Aim and scope

- **“Proof of principle”** new models for forecasting life cycle costs with the CP-140A – only three aircraft

- Nontrivial problem
  - (V. Greenfield and D. Presselin, RAND 2002)
  - (M. Dixon, RAND 2005)
  - (R.A. Pyles, RAND 2003)

Aim and scope

- Fleet replacement – irreversible investment decision under uncertainty

- Future performance/costs unknown – stochastic evolution

- Incorporate effects of an unknown future – stochastic calculus

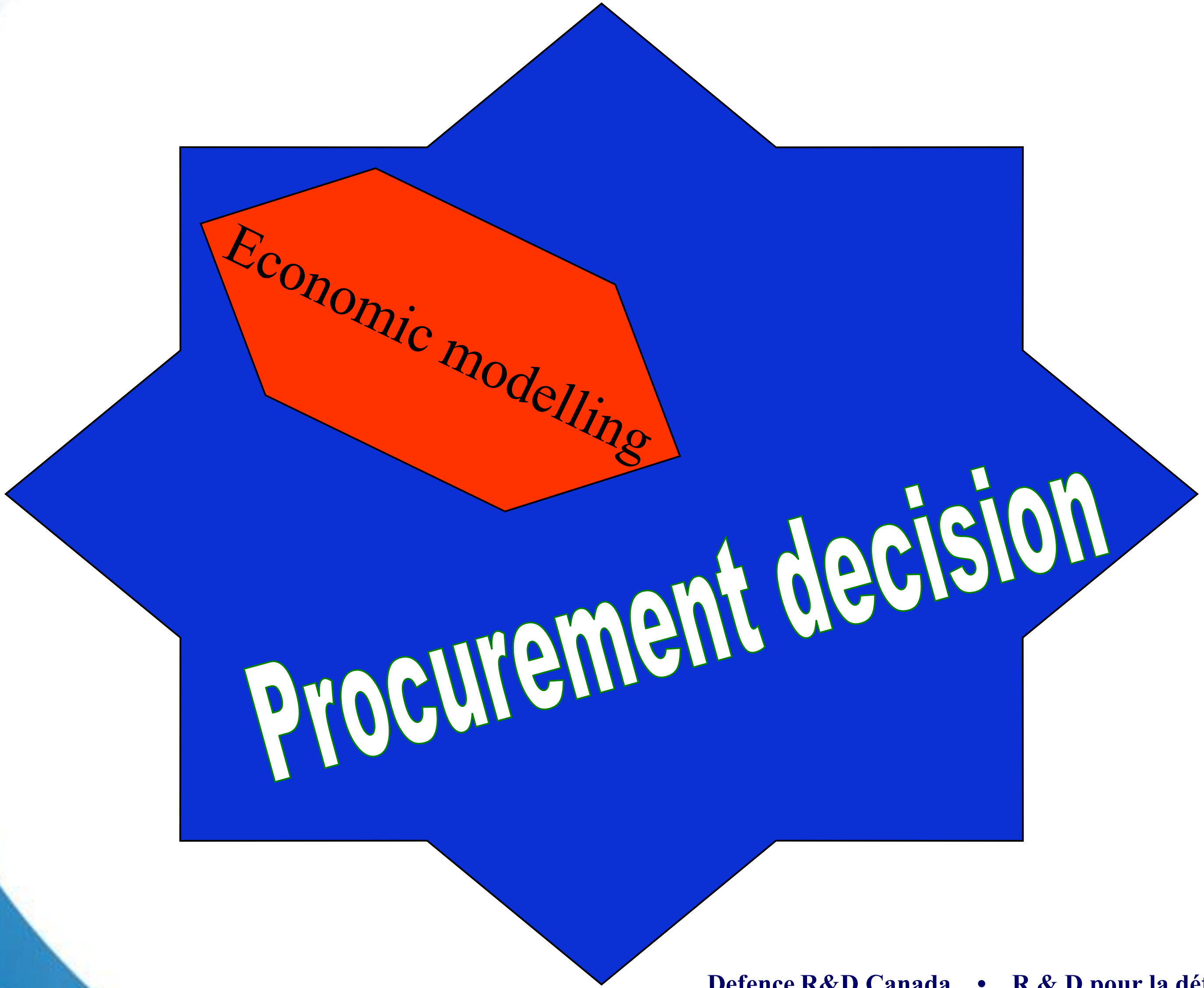
Aim and scope

- **Attach a value to the capacity to wait**

Aim and scope

- Aircraft cost/performance optimization – active area of operations research
  - multilayered optimization software tools
  - Efforts to improve sustainability practices: **DRDC Air Vehicle R&D Program**, Advanced Repair and Life Enhancement – many fronts tackling problems in Life Cycle costing
- Insight can also be gained "Big Picture"

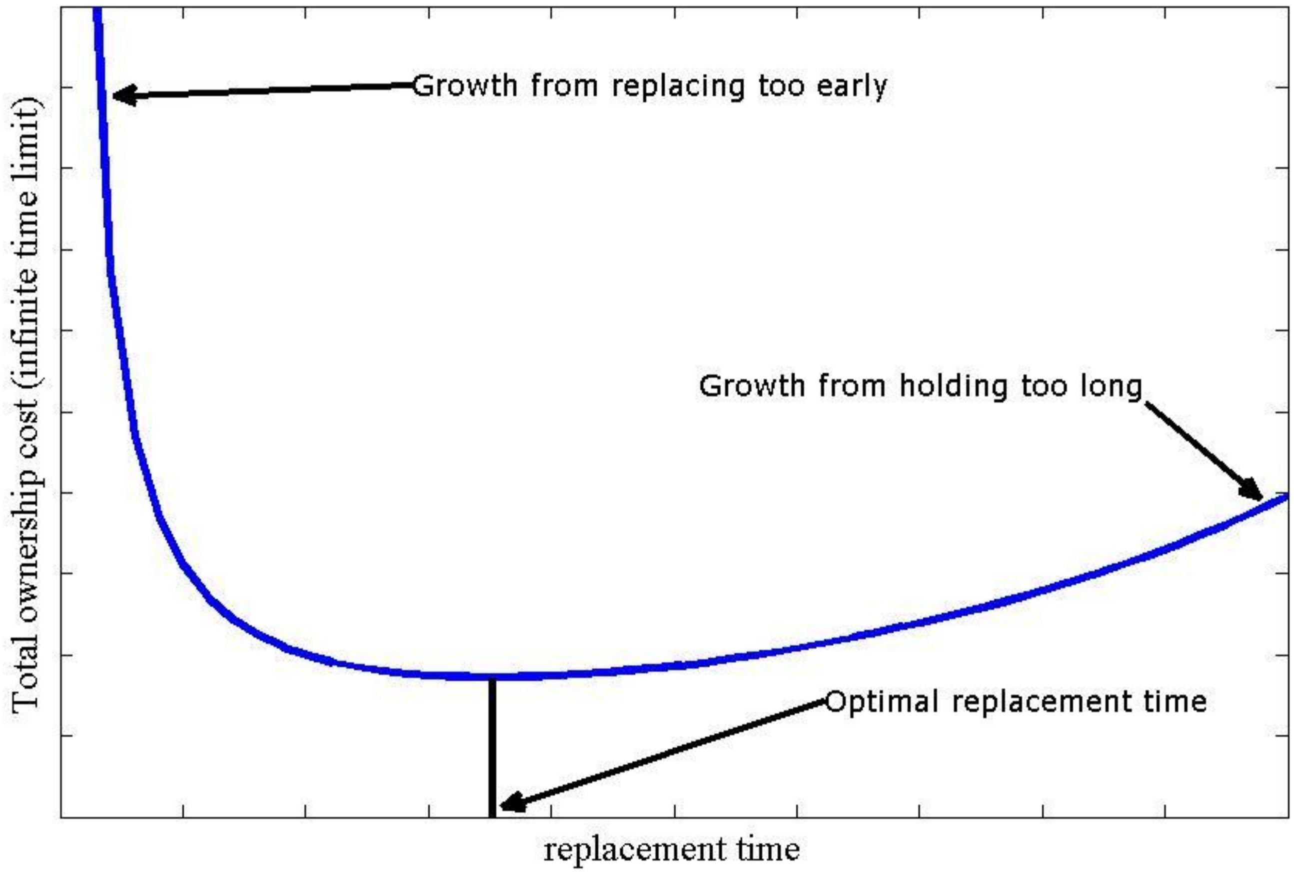
Model assumptions



Model assumptions

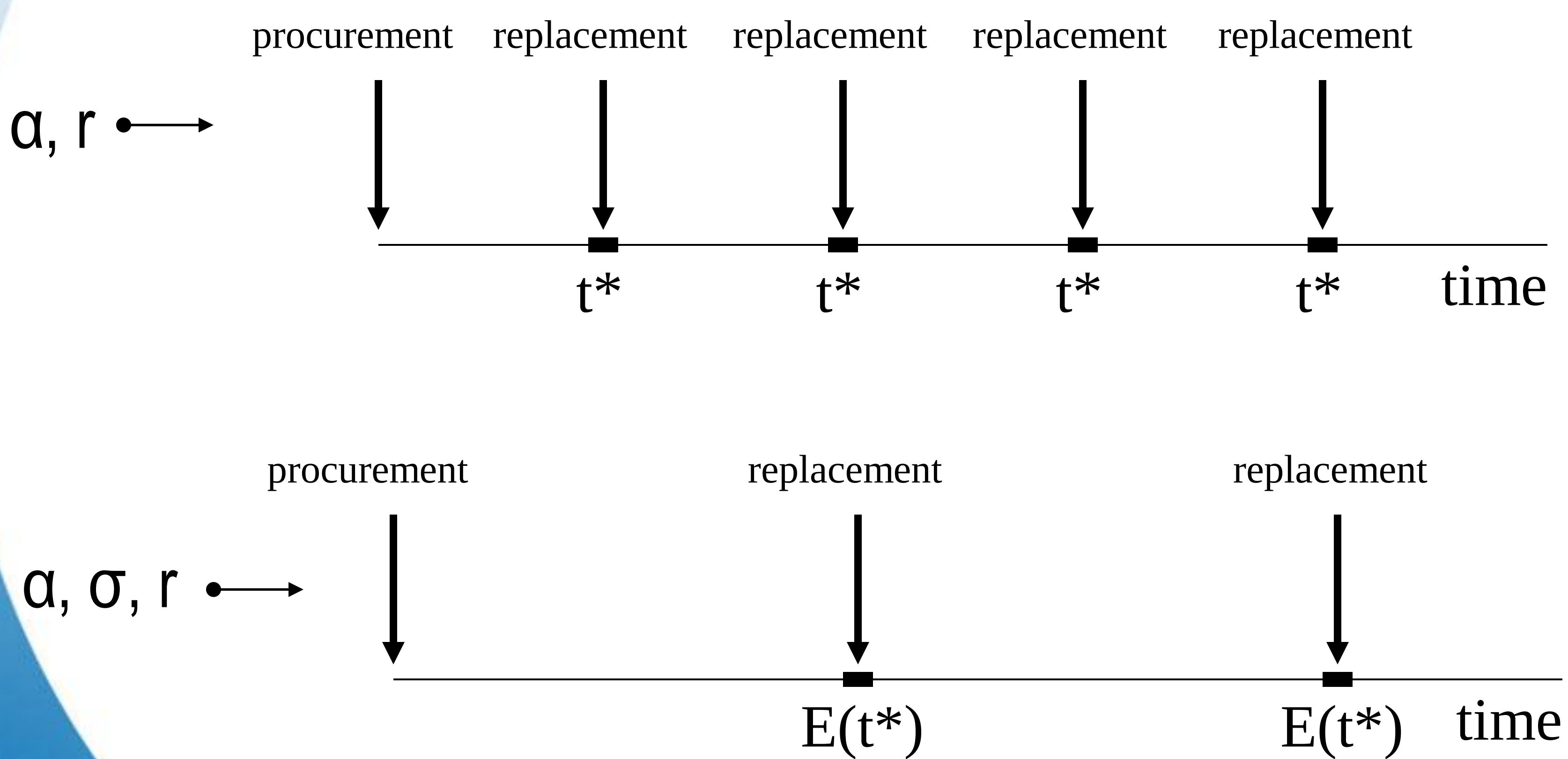
- Replacement choice **not** considered – **like-with-like comparison**
  - Infinite horizon generational replacement – for **specific vehicle**
  - Mathematical model creates like-with-like comparison
  - Discounting process used: 4.40% continuously compounding interest rate
- Costs do not include salaries or squadron support
- **Fleet becomes less resilient to random events with time**
- **Random events occur independently of vehicle life**
- **Age (less resiliency) leads to increasing O&M costs but also to less predictability**

Model assumptions



Model assumptions

Deterministic vs. Stochastic replacement model



Model assumptions

- Stochastic (random) behaviour – model exponential growth in a stochastic background
- **Cost-benefit diffusion model (geometric random walk)** (Greenfield and Presselin, RAND 2002)
- No longer one-to-one relationship between optimal replacement time and annual O&M costs
- Must include a **measure of usefulness** – operational availability (Dixon and Keating, RAND 2003)

Model details

Stochastic case (Ito calculus):

O&M cost evolution:
(geometric Brownian motion)

$$dm_u = \alpha m_u dt + \sigma m_u dW$$

Weiner element

General case (Stochastic differential equation): $dx(t) = a(x,t)dt + b(x,t)dW$

SDE → Partial differential equation for conditional probability density function

Fokker-Planck equation:

$$\frac{\partial}{\partial t} p(x,t|x_0,t_0) = -\frac{\partial}{\partial x}\left(a[x(t),t]p(x,t|x_0,t_0)\right) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(b^2[x(t),t]p(x,t|x_0,t_0)\right)$$

Model details

$$a(m_u^*, m_u) = dt + \left(a(m_u^*, m_u) + \mathrm{E}\left(da(m_u^*, m_u)\right)\right)$$

- Recursive Bellman relation for replacement time

  (Greenfield and Presselin, RAND 2002)

- Resulting Fokker-Planck equation

  - expectation time in a given region

$$\frac{1}{2}\sigma^2 m_u^2 \frac{d^2 a(m_u^*, m_u)}{dm_u^2} + \alpha m \frac{da(m_u^*, m_u)}{dm_u} + 1 = 0$$

Model details

- Recursive Bellman relations for expected replacement time, expected discount factor, and expected net present value (Greenfield and Presselin, RAND 2002)
- Combined relationships yield an expected ownership cost curve which can be minimized to determine the cutoff barrier m_u^* (can impose the cutoff from exogenous policy considerations as well)

Model details

Deterministic replacement time:

$$a(m_u^*, b) = \frac{1}{\alpha} \ln\left(m_u^*/b\right)$$

Stochastic replacement time:

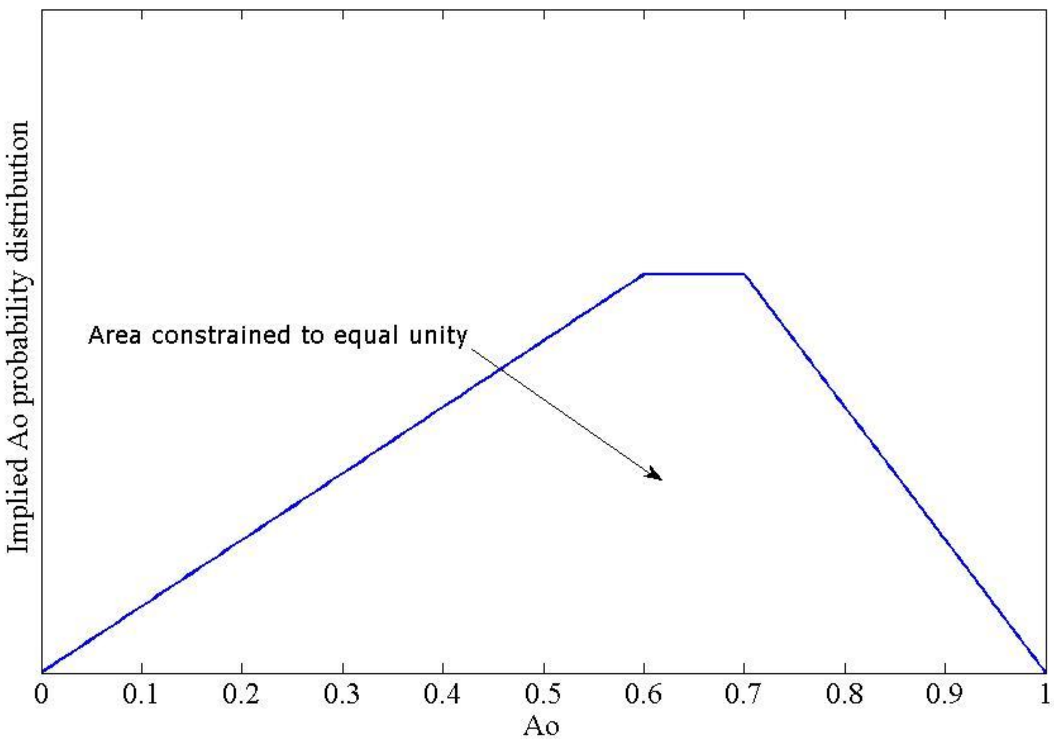
$$a(m_u^*, b) = \frac{\ln\left(m_u^*/b\right)}{\alpha - (1/2)\sigma^2}$$

Model details

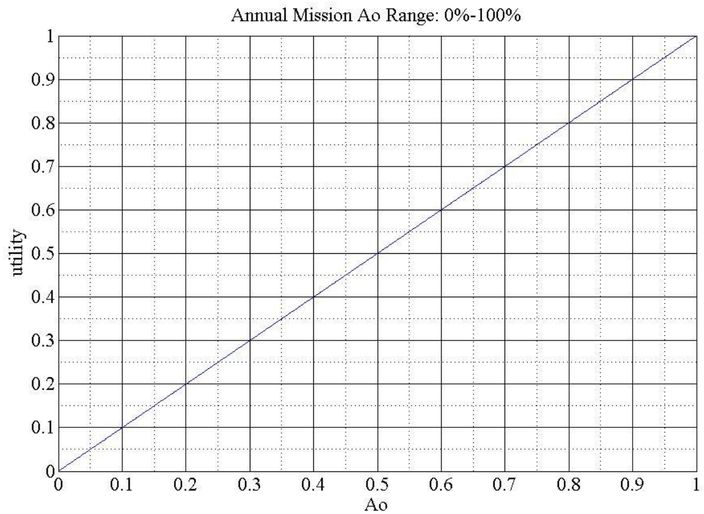
- Model extension: replacement time depends on utility
  - Military requires a performance measure
  - Model requires consensus on the utility measure
  - Operational Availability (A_o) used as proxy for "usefulness"
  - Examine ratio process, approximate by geometric random walk

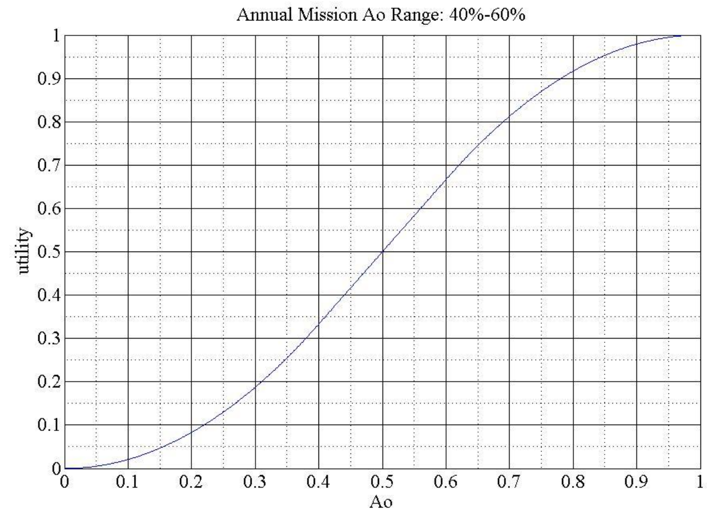
Model details

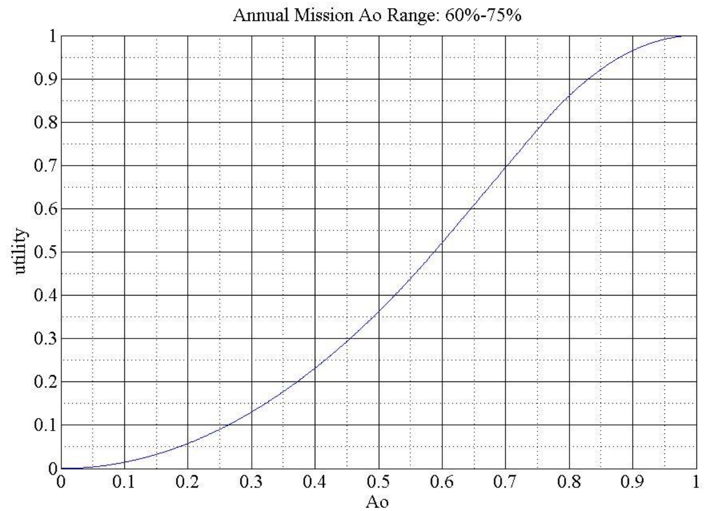
Effectiveness measure: implied utility

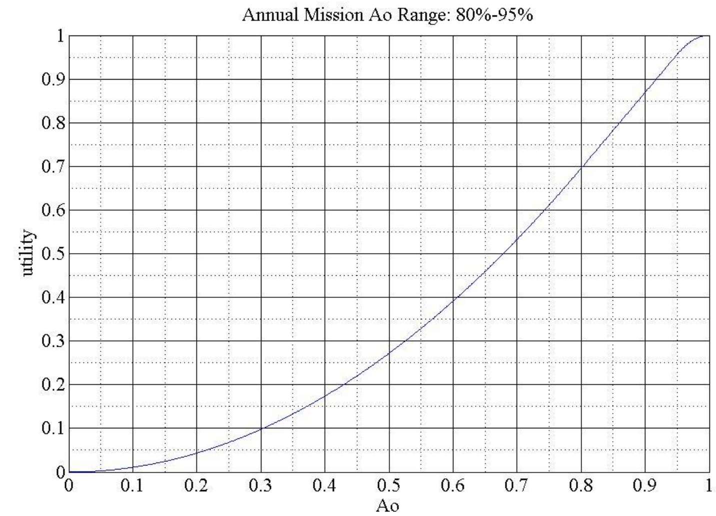


Model details





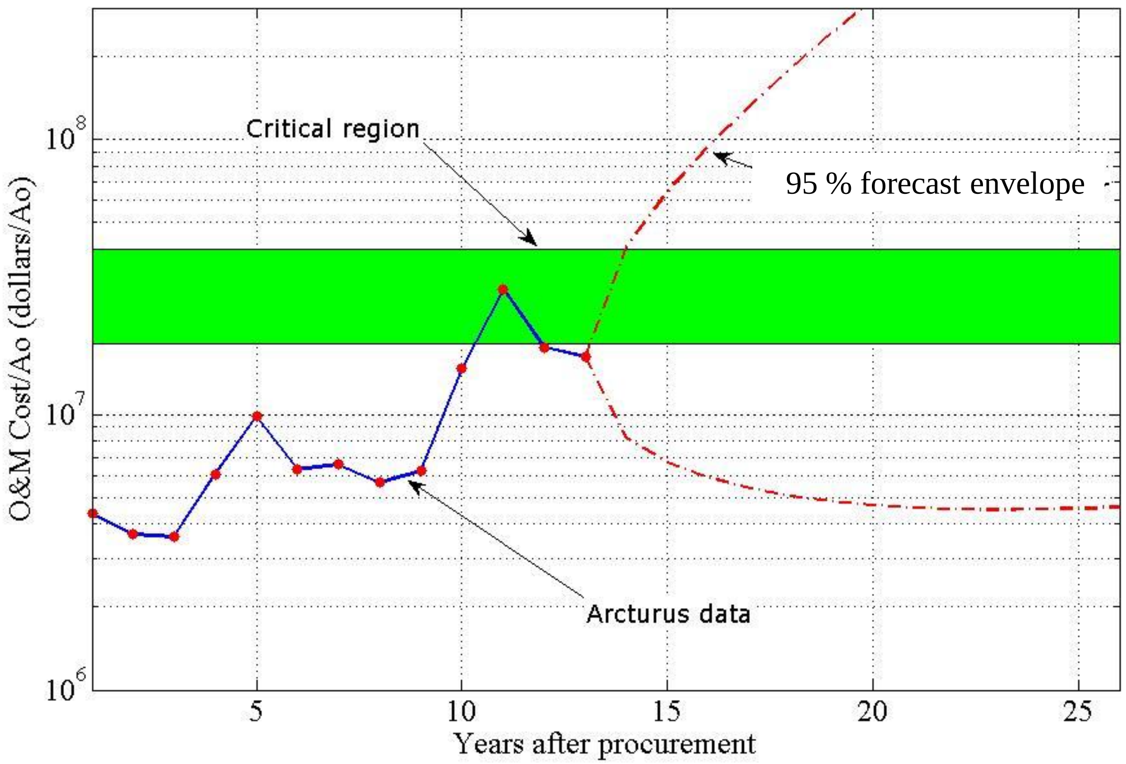




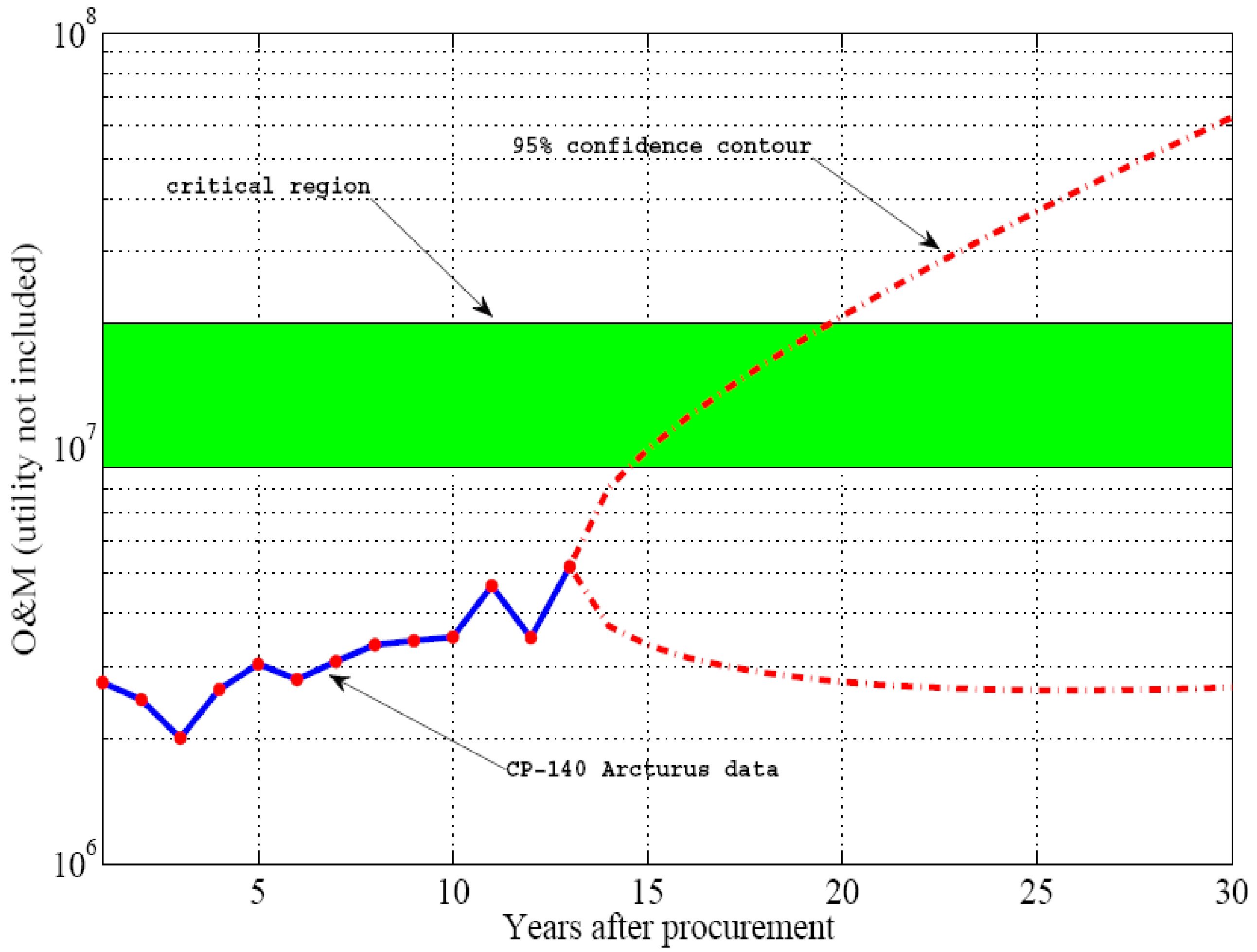
Results

- Theoretically (parameters known with infinite precision), m_u^* is a barrier
- Data has uncertainty – must use error analysis – let the data be the guide
  - Prototype study on system with low amount of data
  - Accommodate uncertainty by "smearing" the barrier with MLE of parameters with error bars
  - Future research: systems with more data, exploit numerical bootstrapping methods (iterated conditioning)
- Data uncertainty: m_u^* becomes a region:
  - *Critical Region:* ***Give the decision maker a decision tool***

Results

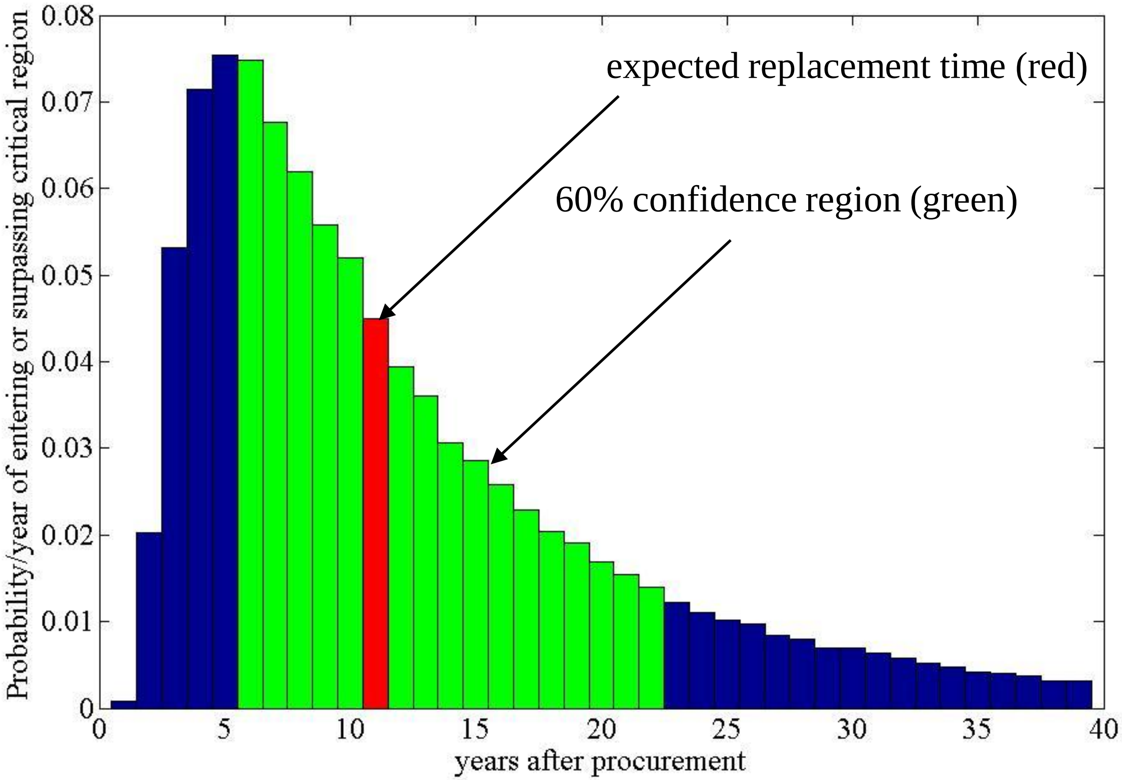


Critical region
95 % forecast envelope
Arcturus data
O&M Cost/Ao (dollars/Ao)
Years after procurement
10^8
10^7
10^6
5
10
15
20
25



95% confidence contour
critical region
CP-140 Arcturus data
O&M (utility not included)
Years after procurement
10^8
10^7
10^6
5
10
15
20
25
30

Results



expected replacement time (red)
60% confidence region (green)
Probability/year of entering or surpassing critical region
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
5
10
15
20
25
30
35
40
years after procurement

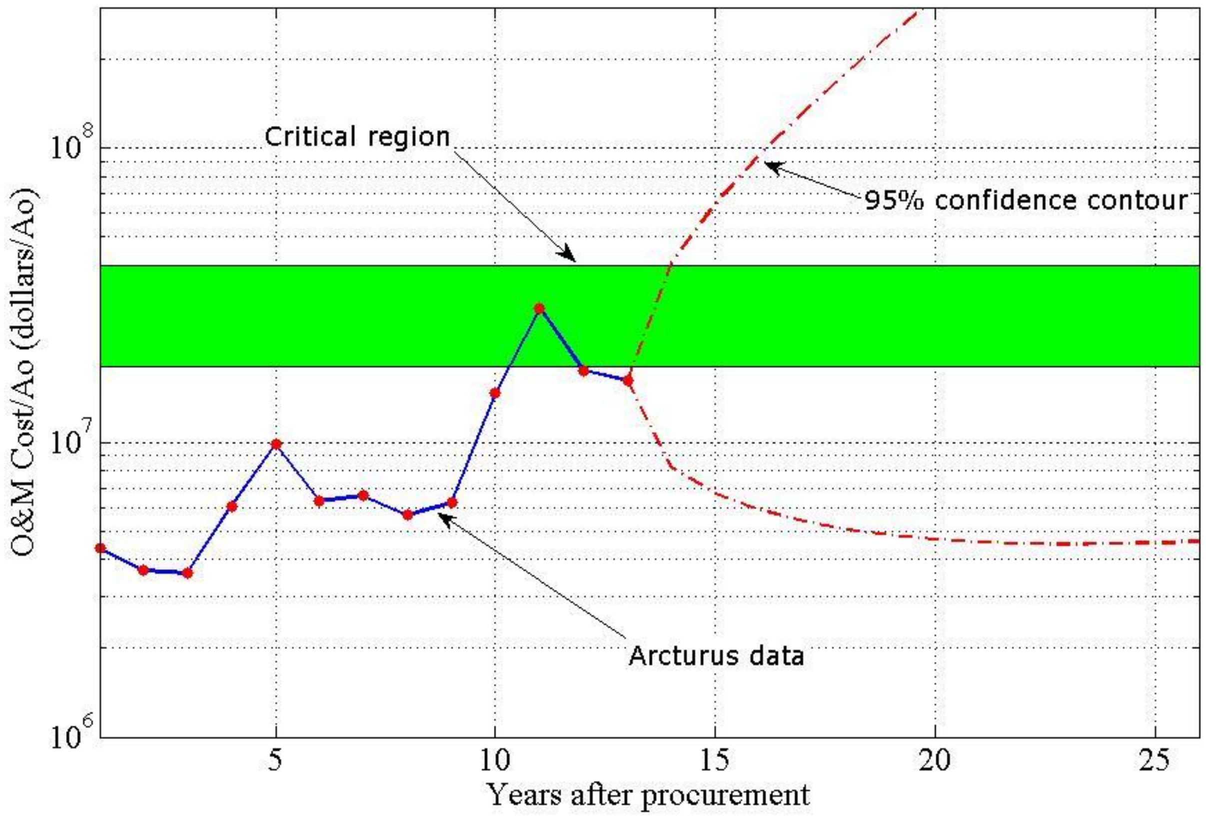
Results

- Expected sojourn time within critical region – help decision makers understand delay costs/benefits

- Generalize the boundary conditions for FPE associated with the expectation time

$$T(\tilde{m}_u) = \frac{1}{k_2}\left[\frac{(\tilde{U}-\tilde{L})(\mathrm{e}^{\frac{-k_2}{k_1}\tilde{U}}-\mathrm{e}^{\frac{-k_2}{k_1}\tilde{m}_u})}{\left(\mathrm{e}^{\frac{-k_2}{k_1}\tilde{L}}-\mathrm{e}^{\frac{-k_2}{k_1}\tilde{U}}\right)}-\tilde{m}_u+\tilde{U}\right]$$

Results



Critical region
95% confidence contour
Arcturus data
10^8
10^7
10^6
O&M Cost/Ao (dollars/Ao)
5
10
15
20
25
Years after procurement

Results

- Expected time has large variance, but can calculate probability of entering critical region from any given time step
- In this example (CP-140A):
  - deterministic “age”: 8 years (with utility),
  - stochastic “age” (with utility): 11 years (60% bounds, 6 years to 22 years)
  - Stochastic “age” (without utility): 23 years
- Use critical region as a decision tool

Summary and conclusions

- Stochastic processes can give longer expectation times for replacement, includes random events
- Once fleet enters the **critical region – procurement process, reset-the clock overhaul, or fleet changes** should be considered
- Can calculate the probability (with utility) of entrance, exit or sojourn times in the critical region from any given time step – help set replacement time
- Model implicitly attaches **value to the capacity to delay** a decision – sunk costs cannot be recovered, waiting has value
- Tool applicable to wide variety of vehicle fleets – **less resilient → higher O&M costs from random events, higher level of unpredictability**

Summary and conclusions

- Model refinements
  - Model the A_o process (stochastic mean reversion, reflected mean reversion, etc.)
  - Correlations in A_o and costing data
  - Departures from geometric random walk (coupled stochastic differential equations)
  - Bootstrap methods (iterated conditioning)

Summary and implications

Possible emergent environment picture: